From: **Michael Berry** asymptotico@bristol.ac.uk
Subject: zeros of erf
Date: March 25, 2018 at 1:32 AM
To: Nicholas Wheeler nwheeler@reed.edu

MB

Dear Nicholas,

Complementary to the vast values of Erf are its zeros. These are visible as the intersections in your figure 1, and nicely illustrated as the phase singularities, shown on the attached picture (one line of Mathematica) with phase colour-coded by Hue.

They lie close to the lines Arg[Erf]=±Pi/4,±3Pi/4, which are the anti-Stokes lines of Erf. Close, but not exactly on the lines, illustrating the asymptotics phenomenon I called 'dominance by subdominant exponentials', and described in my paper 370, where essentially the same example is given. Analytically, the zeros in the first quadrant are approximately

z(n)=r(n)Exp[i th(n)],
r(n)=√(2 Pi (n-1/8)), th(n)=Pi/4+Log[r(n)√(2 Pi)]/(2r(n)^2)

(also given in DLMF 7.13). The other picture show these getting closer to the exact zeros as n increases (the approximate zeros are the black dots). Even the first zero (n=1) is a good approximation.

Nothing original, just fun.

Best wishes, Michael

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From: Nicholas Wheeler <nwheeler@reed.edu>
Sent: 24 March 2018 21:22
To: Michael Berry
Cc: Nicholas Wheeler
Subject: Remarks re Band-limited Superoscillation

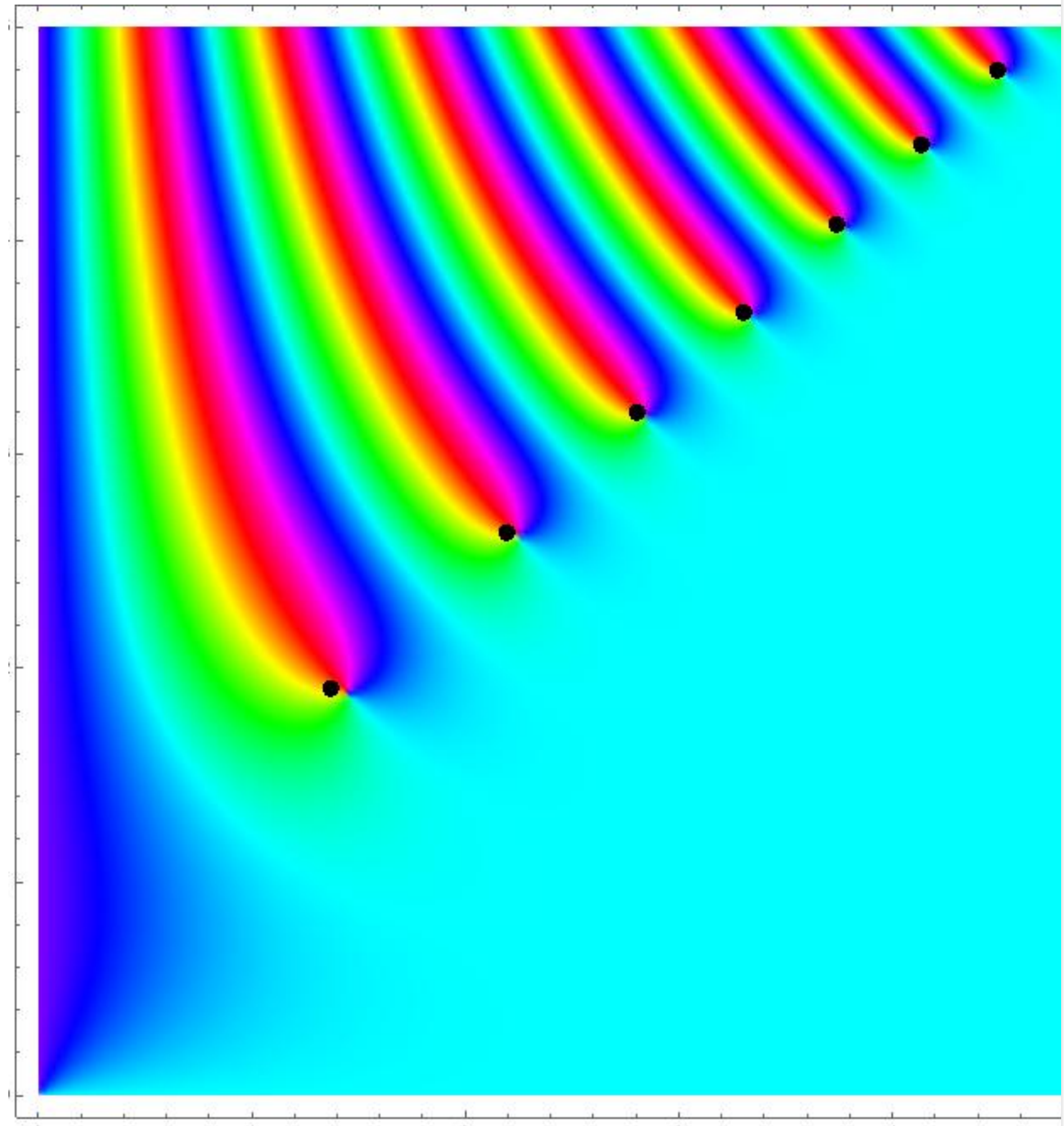
Hello again, Michael

I do not know what interest superoscillations retain among your widely varied interests, but cannot expect you to have more than a passingly courteous interest in what this amateur has to contribute to the subject. But allow myself to share with you some work completed earlier today…because who else can I share it with? Who else could have even slight interest in it?

I was recently motivated to look more closely to the exact solution of your model of band-limited superoscillation. Specifically, I looked to properties of the factor assembled from error functions, which hinge on general properties of error functions with complex arguments. Those are difficult to develop analytically, but can be exposed using the graphic resources of *Mathematica*. I can expect you to have little interest in my text (notes addressed mainly to myself), but think you might possibly have some interesting the accompanying figures.

Regards,

Nicholas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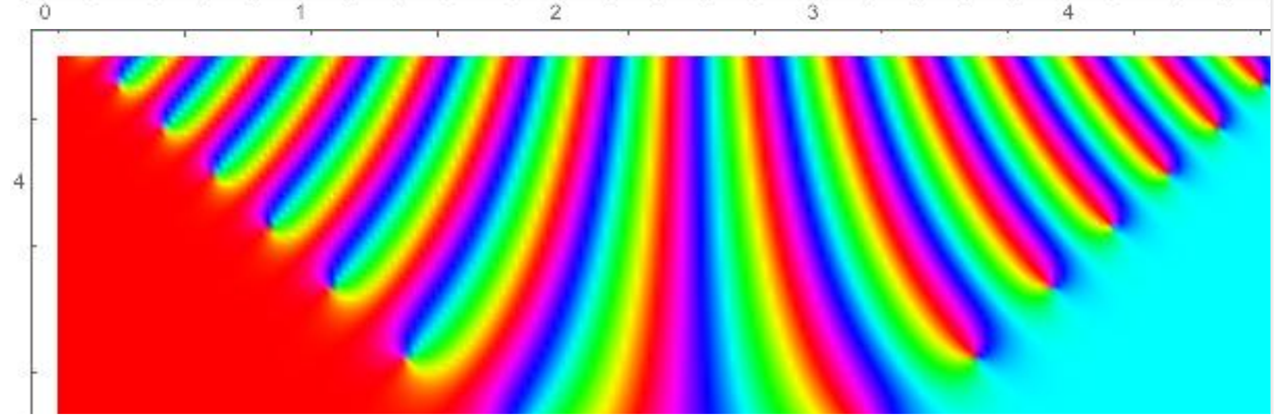
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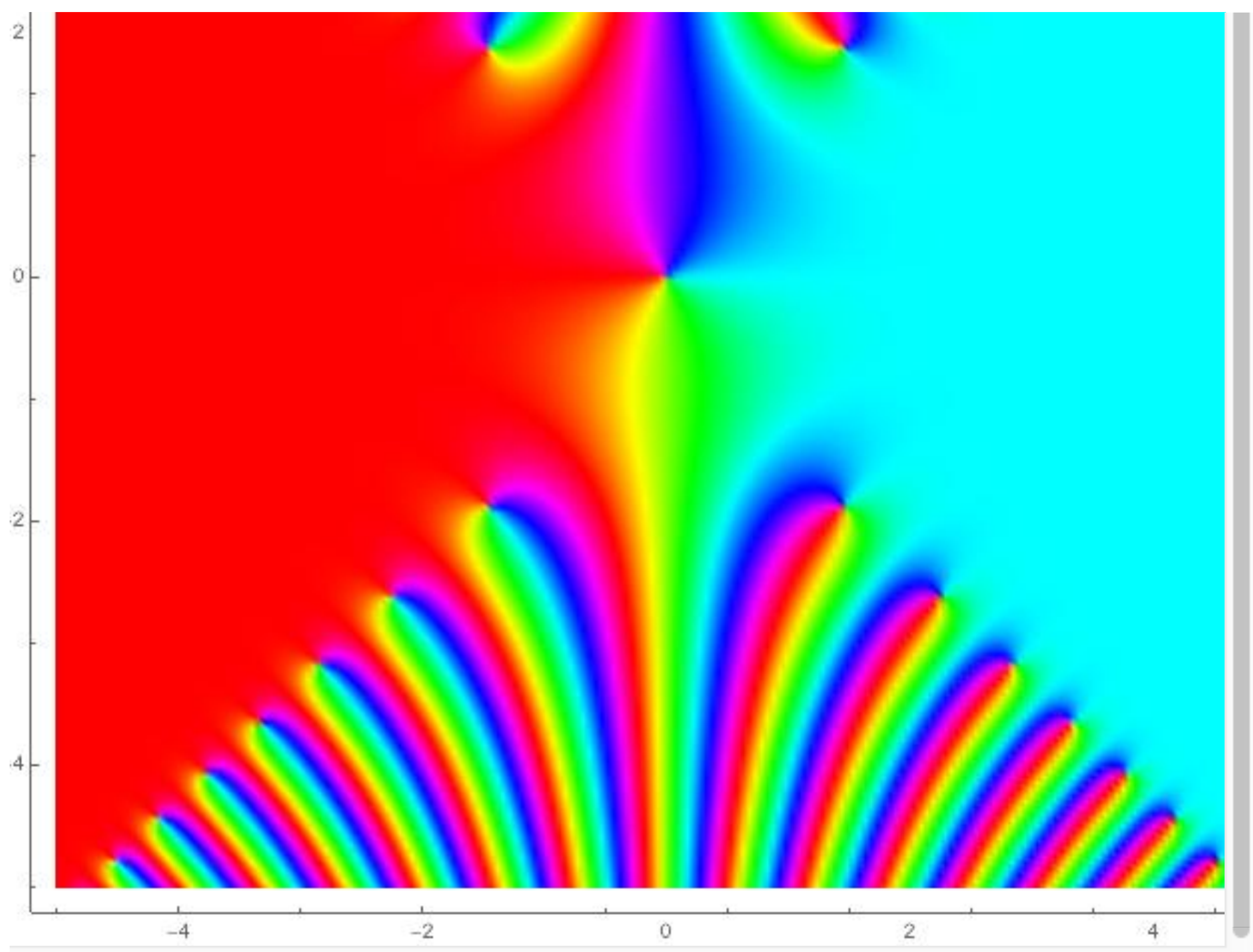

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